

Home Search Collections Journals About Contact us My IOPscience

Model-independent predictions for *n* and d*n*/d ln*k* from a broad class of inflationary models

This article has been downloaded from IOPscience. Please scroll down to see the full text article. 2007 J. Phys. A: Math. Theor. 40 6679 (http://iopscience.iop.org/1751-8121/40/25/S12)

View the table of contents for this issue, or go to the journal homepage for more

Download details: IP Address: 171.66.16.109 The article was downloaded on 03/06/2010 at 05:15

Please note that terms and conditions apply.

J. Phys. A: Math. Theor. 40 (2007) 6679-6687

doi:10.1088/1751-8113/40/25/S12

Model-independent predictions for *n* and $dn/d \ln k$ from a broad class of inflationary models

J A Casas

IFT-UAM/CSIC, 28049 Madrid, Spain

E-mail: alberto.casas@uam.es

Received 4 December 2006, in final form 27 April 2007 Published 6 June 2007 Online at stacks.iop.org/JPhysA/40/6679

Abstract

We explore a well-motivated class of inflationary models from the particle physics point of view: those with a flat tree-level potential, where the radiative corrections cause the slow rolling of the inflaton and the running of the spectral index *n*. This includes typical SUSY inflation models (e.g. D-hybrid inflation). In the small-coupling regime the predictions for the size and running of *n* are remarkably neat and model independent, e.g. $-dn/d \ln k = (n-1)^2 \ll 1$. The fit to WMAP data gives the number of e-folds, N_e , as an output: $N_e = 26^{+30}_{-8}$, which is very encouraging. On the other hand, to account for the WMAP preliminary indication of a running *n* crossing n = 1, we find it extremely natural to incorporate non-renormalizable operators (only the dominant one is relevant), which automatically increase the running as demanded. The analysis is still very model independent and predicts a rapidly changing *n* at the initial scales, which then gets stabilized below n = 1.

PACS numbers: 98.80.Cq, 98.80.Es

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The WMAP data [1, 2] have opened the era of precision observational cosmology. The challenge for the theoretical cosmology and the particle physics is to understand these data and, if possible, to make predictions to be verified by future observations. In this sense, inflation stands as the most successful and promising theoretical scenario. However, the details of the inflationary scheme that reproduces the WMAP (and other) data are still uncertain. Furthermore, a convincing inflationary scheme based on sensible particle physics is still lacking, in spite of interesting developments. (This is why there are so many inflationary models.)

1751-8113/07/256679+09\$30.00 © 2007 IOP Publishing Ltd Printed in the UK

More to the point, the WMAP measurements of the power spectrum of scalar perturbations, P_k , give quite direct information about the shape of the inflationary potential, $V(\phi)$. In the slow-roll approximation [3]:

$$P_k \simeq \frac{1}{24\pi^2\epsilon} \frac{V}{M_p^4} \qquad \text{with} \qquad \epsilon = \frac{1}{2} M_p^2 \left(\frac{V'}{V}\right)^2.$$
 (1)

Let us recall here that the space wave number (sometimes called simply the 'scale') k is related to the inflaton field, ϕ , by

$$\frac{\mathrm{d}\phi}{\mathrm{d}\ln k} = -M_p^2 \frac{V'}{V} = -M_p \sqrt{2\epsilon}.$$
(2)

The variation of P_k with the scale is encoded in the scalar spectral index, n:

$$n-1 = \frac{\mathrm{d}\ln P_k}{\mathrm{d}\ln k} \simeq 2\eta - 6\epsilon$$
 with $\eta = M_p^2 \frac{V''}{V}.$ (3)

Finally, the spectral index itself may change with k:

$$\frac{\mathrm{d}n}{\mathrm{d}\ln k} \simeq -2\xi + 16\epsilon\eta - 24\epsilon^2 \qquad \text{with} \qquad \xi = M_p^4 \frac{V'V'''}{V^2}.\tag{4}$$

Summarizing, observational information about P_k , n and $\frac{dn}{d \ln k}$ gives information about V and its derivatives, V', V'', V''', etc. The WMAP values for n and $\frac{dn}{d \ln k}$ depend on the model assumed for the fit. More precisely [2]:

(a) Assuming a Λ CDM universe [3] with constant *n* (and no tensor perturbations):

$$n = 0.951 \pm 0.016$$
 (68% c.l.). (5)

(b) Assuming a Λ CDM universe with constant $\frac{dn}{d \ln k}$ (and no tensor perturbations):

$$n(k_0) = 1.06 \pm 0.08$$
 (68% c.l.) (6)

$$\frac{\mathrm{d}n}{\mathrm{d}\ln k} = -0.055 \pm 0.030 \quad (68\% \text{ c.l.}). \tag{7}$$

The value of P_k is also slightly dependent on the model, being $P_k \sim (2 \times 10^{-9})$ at $k = k_0 \equiv 0.002 \text{ Mpc}^{-1}$.

What is the impact of these results in particular inflationary models? Using a monomial potential $V \propto \phi^{\alpha}$ (with $\alpha \ge 1$) one gets $n \sim \text{constant}$, in agreement with the model (a) above. Comparing with equation (5), this allows us [1, 2] to put the bound $\alpha < 4$ (95% c.l.) ($\alpha = 4$ is marginally allowed). It is interesting that the data favour renormalizable potentials. Actually, the particular case $V = \frac{1}{2}m^2\phi^2$ works quite well although it requires $\phi_*^2 \sim 200M_p^2$, where the star denotes the 'starting' time of inflation, i.e. the time 50–60 e-folds before the end of inflation.

On the other hand, the indication about a significant running of n, as given by (7), is not compelling, but maybe not irrelevant either [1, 2]; the substantial but not dramatic improvement in the global fit when the running is included does not allow us to be more precise at the moment. If confirmed by future measurements and analyses, it leads us to a very interesting and suggestive situation. Namely, in the region of k tested by WMAP (which corresponds the first ~6–7 e-folds of inflation) n must go from 'blue' (i.e. >1) to 'red' (i.e. <1). As a matter of fact, this is very difficult to achieve in models [4], specially in well-motivated models from the particle physics point of view. (We will come back to this subject in section 3.)

2. A broad class of models

Let us consider a large class of well-motivated models from the particle physics point of view which, however, has not been considered before for these matters in a model-independent way. Namely, consider '*flat tree-level potentials*'¹, i.e.

$$V_{\text{tree}}(\phi) = \rho = \text{constant.}$$
 (8)

Then, the potential derivatives V', V'', V''', \dots arise from the radiative corrections to V. These potentials arise typically in supersymmetric (SUSY) theories: $V_{\text{tree}}^{\text{SUSY}}$ is ordinarily plenty of *accidental* flat directions. Such flatness is broken by radiative corrections since there is no symmetry protecting it. Generically, at 1-loop

$$V(\phi) = \rho + \beta \ln \frac{m(\phi)}{Q},\tag{9}$$

where Q is the renormalization scale (which might have absorbed finite pieces). Note that ρ depends implicitly on Q through its renormalization group equation (RGE) and that the Q-invariance of the effective potential implies $\beta = d\rho/d \ln Q$ at 1-loop. Finally, $m(\phi)$ is the most relevant ϕ -dependent mass (there could be several relevant (and different) masses, which is a complication we ignore for the moment).

The leading-log approximation (which amounts to summing the leading-log contributions to all loops) is implemented in this context by simply taking $Q = m(\phi)$. This choice eliminates the potentially large (and thus dangerous) logs, improving the convergence of the perturbative expansion. Then,

$$V(\phi) \simeq \rho(Q = m(\phi)). \tag{10}$$

In general, one expects $m^2(\phi) = M^2 + c^2\phi^2$, where *M* does not depend on ϕ and *c* is some coupling constant (which depends on *Q* according to its own RGE). We will ignore for the moment the possible presence of the *M* piece, so $Q = c\phi$ and thus

$$\frac{\mathrm{d}Q}{\mathrm{d}\phi} = \frac{Q}{\phi}\alpha = c\alpha,\tag{11}$$

where

$$\frac{1}{\alpha} = 1 - \frac{\beta_c}{c}$$
 and $\beta_c = \frac{\mathrm{d}c}{\mathrm{d}\ln Q}$. (12)

 $(\beta_c \text{ is the conventional } \beta \text{-function of the coupling } c.)$ It is now straightforward to write expressions for the derivatives of V:

$$V' = \alpha \frac{\beta}{\phi},$$

$$V'' = -\alpha \frac{\beta}{\phi^2} \left[1 - \alpha \frac{\dot{\beta}}{\beta} - \alpha^2 \left(\frac{\dot{\beta}_c}{c} - \frac{\beta_c^2}{c^2} \right) \right],$$

$$V''' = \cdots,$$
(13)

where the dots stand for derivatives with respect to $\ln Q$.

R

Note that from equation (2) we can relate the wave number *k* with the scale *Q*:

$$\ln \frac{k_0}{k} \simeq -\frac{\rho}{\beta c^2 M_p^2} (Q_0^2 - Q^2) = -\frac{3H^2}{\beta c^2} (Q_0^2 - Q^2).$$
(14)

Therefore, in these models there is a direct correspondence between slow-roll and RG running. For example, the spectral index, n(k), scans physics at the corresponding (high-energy) scale.

¹ I follow from now on the discussion and results of [5].

Example. An example of such a scenario is the first model of hybrid D-term inflation, proposed by Binétruy and Dvali [6]. The model (BD model from now on) is globally supersymmetric with canonical Kähler potential and with superpotential

$$W = \Phi(\lambda H_{+}H_{-} - \mu^{2}),$$
(15)

where λ and μ^2 are the real constants, and Φ , H_+ and H_- are the chiral superfields. H_{\pm} have charges ± 1 with respect to a U(1) gauge group with gauge coupling g and a Fayet–Iliopoulos term ξ_D . The associated tree-level scalar potential is given by $V_0 = V_F + V_D$, with

$$V_F = |\lambda H_+ H_- - \mu^2|^2 + \lambda^2 (|H_-|^2 + |H_+|^2) |\Phi|^2,$$

$$V_D = \frac{g^2}{2} (|H_+|^2 - |H_-|^2 + \xi_D)^2,$$
(16)

(the fields in these expressions are the scalar components of the chiral superfields). The global minimum of the potential is supersymmetric (i.e. $V_F = 0, V_D = 0$) and occurs at $\Phi = 0, \langle H_{\pm} \rangle \neq 0$. But for large enough $|\Phi|$

$$\langle H_{\pm} \rangle = 0, \qquad V_0 \equiv \rho = \mu^4 + \frac{1}{2}g^2 \xi_D^2.$$
 (17)

After radiative corrections, the 1-loop potential takes the form (9) with ρ given by (17), $m(\phi) = \frac{1}{2}\lambda^2\phi^2$ and $\beta = \frac{1}{8\pi^2}(g^4\xi_D^2 + \lambda^2\mu^4)$.

3. Small-coupling regime

In the regime of very small coupling constants one has $\beta_c/c \ll 1$ and thus $\alpha \simeq 1$. Correspondingly, all the previous equations (11)–(13) become greatly simplified and (even more importantly) model independent. In particular,

$$V' \simeq \frac{\beta}{\phi}, \qquad V'' \simeq -\frac{\beta}{\phi^2}, \qquad V''' \simeq 2\frac{\beta}{\phi^3}, \quad \text{etc}$$
 (18)

and hence

$$\epsilon \simeq \frac{1}{2} \frac{M_p^2}{\phi^2} \left(\frac{\beta}{\rho}\right)^2, \qquad \eta \simeq -\frac{M_p^2}{\phi^2} \frac{\beta}{\rho} \simeq -2 \left(\frac{\beta}{\rho}\right)^{-1} \epsilon, \qquad \xi \simeq 2\eta^2.$$
(19)

Normally $\epsilon \ll \eta$, and thus $n - 1 \simeq 2\eta$. From the previous expressions it is straightforward to get model-independent predictions for physical observables. In particular, the running of *n* with $\ln k$ is given by

$$\frac{\mathrm{d}n}{\ln k} \simeq -2\xi = -(n-1)^2.$$
(20)

Consequently, $dn/d \ln k$ is negative, as suggested by observation, though its value tends to be quite small. In fact, the sign of n - 1 cannot change along the inflationary process, which would contradict the WMAP indication of n crossing n = 1, if it eventually gets confirmed. In addition sign $(n - 1) = -\text{sign}(V') = -\text{sign}(\beta)$. Usually β (and thus V') is positive and therefore we naturally expect n < 1 but this is not mandatory.

The number of e-folds, N_e , since t_* until the end of inflation can be easily computed plugging ϵ , as given by equation (19) in the usual expression $N_e \simeq \frac{1}{M_e} \int_{\phi_{end}}^{\phi} \frac{1}{\sqrt{2\epsilon}} d\phi$,

$$N_e \simeq -\frac{1}{2\eta(\phi_*)} \simeq \frac{1}{1 - n_*}.$$
 (21)

This can be used to write equation (20) in an integrated form as

$$n = 1 - \frac{1}{N_e - \ln(k/k_*)}.$$
(22)

Note that N_e is the only independent parameter in this equation.

Now, we can wonder how well do the previous expressions fit the WMAP results. Using the COSMOMC program to perform the fit, one gets the following results² [7]:

- 68% c.l. $\longrightarrow n(k_0 = 0.05 \text{ Mpc}^{-1}) = 0.95 \pm 0.017 \implies N_e^0 = 20_{-5}^{+10}$ 95% c.l. $\longrightarrow n(k_0 = 0.05 \text{ Mpc}^{-1}) = 0.95_{-0035}^{+0.030} \implies N_e^0 = 20_{-8}^{+30}$

Here N_e^0 represents the number of e-folds from $k_0 = 0.05$ till the end of inflation. Thus $N_e^* \simeq N_e^0 + 6$ e-folds. Note that N_e has been considered just a parameter of the fit, so we could have obtained $N_e \sim 1$ or $\sim 10^3$ or ~ -20 . Remarkably, however, it turns out to be consistent with the desired $N_e^* = 50-60$ at 95% c.l. (for a complementary approach see [8]). We find these results very nice and non-trivial. We can also allow for tensor perturbations (which means a non-negligible ϵ parameter, since $P_t(k) = 16\epsilon P_k$). Then we cannot ignore the ϵ contribution in equations (3), (4). This is still under study.

In summary, the results obtained by considering small coupling in scenarios with flat treelevel potential are both very model independent and non-trivially consistent with the WMAP results. They imply in particular $\frac{dn}{d \ln k} \simeq -(n-1)^2 \ll 1$. However, they could not explain a running n(k) crossing n = 1. Therefore, if the WMAP indication $\frac{dn}{d \ln k} \sim -0.055$ gets finally confirmed, these would be excluded . . . but not only them! To cook up inflationary models able to account for that fact has proved to be a difficult task, and the few existing examples are rather artificial, lacking physical motivation.

Hence, it would be nice if the models considered above could be modified in a wellmotivated way, so that they could account for the $\frac{dn}{d \ln k} \sim -0.055$ preliminary indication.

4. No-so-small-coupling regime

If the β -functions are positive (as usual, see the above discussion), couplings grow in the ultraviolet and there will be a scale where the second and third terms within the square brackets in (13) compete with the first one. Since they naturally have the opposite sign, one can expect that at sufficiently high scales (which means initial stages of inflation) the sign of η , and thus n-1, may get positive, which therefore could account for the preliminary indication of WMAP.

As a matter of fact, we find this behaviour absolutely natural and even unavoidable in this regime. However, the analysis becomes model dependent (there are new parameters in the game as $\dot{\beta}$, β_c , $\dot{\beta}_c$). Furthermore, reproducing the WMAP preliminary indication on the running spectral index and the other physical requirements at the same time (in particular, a sufficient number of e-folds) is not a trivial matter.

We have explored this no-so-small-coupling regime in the BD model, depicted at the end of section 2, finding analytically the constraint[5]

$$-\left(N_e^0\right)^2 \times \left.\frac{\mathrm{d}n}{\mathrm{d}\ln k}\right|_{Q_0} \simeq 1.1,\tag{23}$$

which makes it impossible to fulfil $dn/d \ln k|_{k_0} = \mathcal{O}(-0.05)$ and $N_e^0 \sim \mathcal{O}(50)$ simultaneously. (A similar kind of bounds was obtained later in [9] in a more general context. For more related work on this subject see [10].)

 $^{^2}$ On the technical side, let us mention that we have used six chains, examining $\sim 10^5$ samples of parameters each. The number of steps accumulated in each chain is about 3×10^4 . The parameters fitted are seven: $\Omega_B h^2$, $\Omega_{CDM} h^2$, θ , τ (I followed the notation of WMAP) and the three parameters of the model: $P_k(k_*), N_e, \epsilon$ (the latter only if the tensor modes are included, as commented below). The convergence criterion used is the standard Raftery and Lewis one.



Figure 1. Scalar spectral index as a function of $\log_{10}[k(\text{Mpc}^{-1})]$ for the inflaton potential of the BD model for two different choices of parameters.

The previous behaviour is illustrated by figure 1 which gives the spectral index n as a function of the scale wave number k for two different choices of parameters: one to get right N_e and the other to get right $dn/d \ln k$.

Of course, one cannot exclude that other tree-level-flat models are able to reproduce such running spectral index without the above limitation for the number of e-folds. To achieve this one needs sizeable η at high scale, so that the spectral index runs appreciably in the first stages of inflation, and small ϵ at lower scales, so that a sufficient number of e-folds is produced. This implies that β (and thus the relevant coupling constants) must evolve from sizeable values at high scale to substantially smaller values at lower scales. In this sense, the crossing of some threshold of new physics along the inflationary course can help, though the analysis for the BD model shows that this is not enough to solve the problem. Fortunately, there is a simpler and better alternative, which we discuss next.

5. Non-renormalizable operators (NRO)

Suppose there is a scale of new physics, M, higher than the scales relevant to inflation (i.e. $\phi^2 \ll M^2$). In general, this physics will show up at low scales as non-renormalizable operators (NROs) suppressed by inverse powers of M. Due to the suppression factor, the impact of the NRO in the physics at low scales is normally very small. However, if the NRO has characteristics not shared by the low-energy physics, its effect may be significant (as happens with higher dimension operators that mediate proton decay or give a Majorana mass to the neutrinos). In our case, the new physics does not need to respect the accidental flat directions of the effective theory. Thus one expects

$$V(\phi) = \rho + \beta \ln \frac{m(\phi)}{Q} + \phi^4 \frac{\phi^{2N}}{M^{2N}}.$$
(24)

The first two terms just correspond to the generic 1-loop potential we have discussed in previous sections. In the small-coupling regime, as in subsection 2.1, we take β as a constant.



Figure 2. Example of inflaton effective potential (normalized to ρ) with a NRO as in equation (24) for N = 9. The star marks ϕ_* and the circle, ϕ_0 .

The last term in (24) is a non-renormalizable operator (NRO) left in the low-energy theory after integrating out some unspecified physics at the high scale M. This scale absorbs any possible coupling in front of the operator. Of course $V(\phi)$ may contain other NROs of different order. Here we assume that the one shown in equation (24) is the lowest order one, and thus the dominant. The sign and power we have assumed for this NRO are convenient to guarantee the stability of the potential. Note also that an even power for this operator is what one expects generically in supersymmetric theories (an explicit example of this is given in [5]). Apart from this, the potential (24) is completely general, and therefore the analysis is essentially model independent. By trivial inspection of the derivatives of $V(\phi)$ with respect to ϕ ,

$$V'(\phi) = \frac{\beta}{\phi} + 2(N+2)\phi^3 \frac{\phi^{2N}}{M^{2N}},$$

$$V''(\phi) = -\frac{\beta}{\phi^2} + 2(N+2)(2N+3)\phi^2 \frac{\phi^{2N}}{M^{2N}},$$

$$V'''(\phi) = 2\frac{\beta}{\phi^3} + 4(N+2)(2N+3)(N+1)\phi \frac{\phi^{2N}}{M^{2N}},$$
(25)

we realize that the NRO can have a significant impact on inflation when the small number $(\phi/M)^{2N}$ is comparable in size to β/ϕ^4 (which is also quite small). It is also immediate to realize from (25) that, for sufficiently large ϕ , the higher derivatives V'', V''' (and thus η, ξ) can receive a large contribution from the NRO while the contribution to V' (and thus ϵ) is much less significant, thanks to the additional (2N + 3) and (2N + 3)(2N + 2) factors in V'', V'''. This is precisely what we need to modify the spectral, n, and its running with k at the initial scales, without changing ϵ (and hence N_e) significantly.

This is illustrated in figure 2, which shows the effective potential (24) as a function of ϕ/M_p for a particular choice of the parameters. It is easy to show [5] that larger values of N lead to stronger running of n. In our case we have taken N = 9, which is rather large but perfectly possible in the context of SUSY and string constructions (for details see [5]). The star and the circle denote ϕ_* and ϕ_0 , respectively. Note how ϕ_* is below the range where the NRO starts to be important for $V'(\phi)$ (but not for $V''(\phi)$). The corresponding slow-roll parameters



Figure 3. Slow-roll parameters (upper plot) and scalar spectral index as a function of scale in Mpc^{-1} (lower plot) for the inflaton potential of figure 2.

are shown in the upper plot in figure 3 as a function of ϕ/M_p . Finally, the lower plot of figure 3 gives the scalar spectral index as a function of k, which has the desired behaviour, with $dn/d \ln k|_* \simeq -0.03$ and $N_e \simeq 50$. (There are many more successful examples.)

In general, an accurate approximation for the spectral index is

$$n = 1 - \frac{1}{N_e - \ln(k/k_*)} - \frac{N_e}{(N+1)} \left(\frac{\mathrm{d}n}{\mathrm{d}\ln k} \Big|_* + \frac{1}{N_e^2} \right) \left(1 - \frac{1}{N_e} \ln \frac{k}{k_*} \right)^{N+1}$$
(26)

(this corresponds to equation (77) in [5], after some algebra to simplify the expression). Note that *n* depends on more parameters than before introducing the NRO (now it depends on N_e , $dn/d \ln k|_*$ and *N*) but it is still quite model independent.

6. Conclusions

We have considered a broad and well-motivated class of models, defined by having flat treelevel potential, which is typical of SUSY scenarios. In these models V', V'', \ldots arise from radiative corrections.

In the small-coupling regime the predictions are very clean and model independent, e.g. $\frac{dn}{d \ln k} \simeq -2\xi = -(n-1)^2$, $n = 1 - \frac{1}{N_e - \ln(k/k_*)}$. The fit to the WMAP data gives $N_e = 26^{+30}_{-8}$ (95% c.l.), which is remarkably consistent with a succesful inflation.

However, this regime is not consistent with a running n(k) crossing n = 1. Therefore, if the WMAP indication in this sense got finally confirmed, these models (and most inflation models) would be excluded. On the other hand, we have discussed how adding a non-renormalizable operator (NRO) that spoils the accidental tree-level flatness helps a lot to get a sizeable running of n at the initial stages of inflation. The analysis can be still done in an essentially model-independent way, and we have explicitly shown how the WMAP can be accounted for in this way.

Acknowledgments

I thank Guillermo Ballesteros and José Ramón Espinosa, and more recently Roberto Ruiz de Austri and Roberto Trotta, for the enjoyable collaboration leading to the work reported here. This work was supported by the Spanish Ministry of Education and Science through a MEC project (FPA 2004-02015) and by a Comunidad de Madrid project (HEPHACOS; P-ESP-00346).

References

- [1] Peiris H V et al 2003 Astrophys. J. Suppl. 148 213 (Preprint astro-ph/0302225)
- [2] Spergel D N et al (WMAP Collaboration) 2006 Preprint astro-ph/0603449
- [3] For a review, see, e.g., Lyth D H and Riotto A 1999 Phys. Rep. 314 1 (Preprint hep-ph/9807278) Alabidi L and Lyth D H 2005 Preprint astro-ph/0510441
- [4] Chung D J H, Shiu G and Trodden M 2003 Phys. Rev. D 68 063501 (Preprint astro-ph/0305193)
- [5] Ballesteros G, Casas J A and Espinosa J R 2006 J. Cosmol. Astropart. Phys. JCAP03(2006)001 (Preprint hep-ph/0601134)
- [6] Binétruy P and Dvali G R 1996 Phys. Lett. B 388 241 (Preprint hep-ph/9606342)
- [7] Ballesteros G, Casas J A, Espinosa J R, Ruiz de Austri R and Trotta R in preparation
- [8] Peiris H and Easther R 2006 J. Cosmol. Astropart. Phys. JCAP10(2006)017 (Preprint astro-ph/0609003)
- [9] Easther R and Peiris H 2006 J. Cosmol. Astropart. Phys. JCAP09(2006)010 (Preprint astro-ph/0604214)
- [10] Leach S M and Liddle A R 2003 Phys. Rev. D 68 123508 (Preprint astro-ph/0306305)
 Leach S M and Liddle A R 2003 Mon. Not. R. Astron. Soc. 341 1151 (Preprint astro-ph/0207213)
 Peiris H and Easther R 2006 J. Cosmol. Astropart. Phys. JCAP07(2006)002 (Preprint astro-ph/0603587)
 Finelli F, Rianna M and Mandolesi N 2006 J. Cosmol. Astropart. Phys. JCAP12(2006)006 (Preprint astro-ph/0608277)

Cline J M and Hoi L 2006 J. Cosmol. Astropart. Phys. JCAP06(2006)007 (Preprint astro-ph/0603403) Chung D J H and Enea Romano A 2006 Phys. Rev. D **73** 103510 (Preprint astro-ph/0508411)